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"Convergence of MCMC and Loopy Belief Propagation in the Tree Uniqueness Region for the Hard-Core Model"

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Abstract: We study the hard-core model defined on independent sets of an input graph where the independent sets are weighted by a parameter $\lambda>0$. For constant Δ, previous work of Weitz (2006) established an FPTAS for the partition function for graphs of maximum degree Δ when $\lambda<\lambda_c(\Delta)$. The threshold $\lambda_c(\Delta)$ is the critical point for the phase transition for uniqueness/non-uniqueness on the infinite Δ-regular trees. Sly (2010) showed that there is no FPRAS, unless NP=RP, when $\lambda>\lambda_c(\Delta)$. The running time of Weitz's algorithm is exponential in $\log(\Delta)$. Here we present an FPRAS for the partition function whose running time is $O(n^2)$. We analyze the simple single-site Glauber dynamics for sampling from the associated Gibbs distribution. We prove there exists a constant Δ_0 such that for all graphs with maximum degree $\Delta \geq \Delta_0$ and girth ≥ 7, the mixing time of the Glauber dynamics is $O(n \log(n))$ when $\lambda<\lambda_c(\Delta)$. Our work complements that of Weitz which applies for constant Δ whereas our work applies for all $\Delta \geq \Delta_0$.

We utilize loopy BP (belief propagation), a widely-used inference algorithm. A novel aspect of our work is using the principal eigenvector for the BP operator to design a distance function which contracts in expectation for pairs of states that behave like the BP fixed point. We also prove that the Glauber dynamics behaves locally like loopy BP. As a byproduct we obtain that the Glauber dynamics converges, after a short burn-in period, close to the BP fixed point, and this implies that the fixed point of loopy BP is a close approximation to the Gibbs distribution. Using these connections we establish that loopy BP quickly converges to the Gibbs distribution when the girth is ≥ 6 and $\lambda<\lambda_c(\Delta)$.

This is a joint work with Tom Hayes, Daniel Stefankovic, Eric Vigoda and Yitong Yin.

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